2016 Invitational Round

1. In $\triangle ABC$, $\angle A = \angle B = 2\angle C$. Find $\angle C$.

2. $\triangle ABC$ has side lengths $3, 4$, and 5 with incenter I. Let the reflections of I across BC, CA, and AB be D, E, and F. Find the area of hexagon $AECDBF$.

3. In $\triangle ABC$, $\angle A = 80°$. Denote the orthocenter (intersection of the altitudes) of $\triangle ABC$ as H. Find the measure of $\angle BHC$.

4. Right triangle ABC has side lengths $AB = 15$, $BC = 20$, and $AC = 25$. A circle centered at B is tangent to AC. Compute the area of the circle that is inside the triangle.

5. Square $ABCD$ has side length 4. Let P be a point outside of $ABCD$ such that PA and PD intersects side BC at E and F. If $EF = 3$, find the area of $\triangle PAD$.

6. Daniel is trying to fit two identical circular cookies into a square with side length 8 such that his two cookies must be completely inside of the square and cannot overlap. Find the largest radius that the two cookies can have.

7. $\triangle ABC$ has side lengths $AB = 3, BC = 4$, and $CA = 5$. Let K be the intersection of the altitude from B to AC and the perpendicular bisector of segment AB. Let the circumcenter of $\triangle ABC$ be O. Find the length of KO.

8. Jeffery the Jant is sitting on the circumference of the bottom face of a cylindrical can with height 80π and radius 12. A magical bread crumb lies diametrically opposite of Jeffery, but on the top face of the cylinder. Jeffery can only walk along the surface of the cylinder (not through the bases), and the crumb will disappear if Jeffery does not walk around the cylinder completely at least 2 times. Find the shortest distance Jeffery can walk to get the bread crumb.

9. Let circle ω have diameter AB with length 5. A line l is drawn outside of ω perpendicular to AB such that l is closer to B than to A. Let C be chosen on ω such that $AC = 4$. Let AB and AC intersect l at D and E respectively. Given that $CE = 6$, compute $\frac{BE}{CD}$.

10. Square $ABCD$ has side length 3. A point E lies on side $\overline{AB}$ such that $\angle ADE = 30$, and a point F lies outside the square such that $\triangle EBF$ is equilateral. The circumcircle of $\triangle FBD$ intersects the square at two more points M and N. Compute MN.