Set 1 - 10 points each

1. Jeffery the Jangaroo hops exactly once around the perimeter of a triangle with side lengths $2016, 2017$, and 2. How far is Jeffery from where he started?

2. A trapezoid $ABCD$ has $AB \parallel CD$ and $AB \perp AD$. If $AB = 7, CD = 12$, and $AD = 11$, find the length of BC.

3. $ABCDEF$ is a regular hexagon with side length 6. Find the area of $\triangle ACE$.

Set 2 - 11 points each

4. A rectangle has side lengths 4 and 5. Its diagonal has length x. Find the area of a square with side length x.

5. Given 3 fixed points A, B, and C, how many different points P are there such that the points A, B, C, and P form a parallelogram?

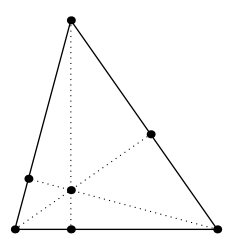
6. Consider a square $ABCD$ and a point P. If the area of $\triangle ABP$ is 3 and the area of $\triangle CDP$ is 15, find the sum of all possible areas of $ABCD$.

Set 3 - 12 points each

7. A right triangle has side lengths $3x + 1, 4x + 1$, and $5x + 1$. Find x.

8. Four points A, B, C, D are randomly chosen in a plane. Find the probability that segment AB intersects segment CD.

9. The figure below shows 7 points: the vertices of a triangle, the feet of its altitudes, and its orthocenter. How many different circles are there that pass through at least 4 of the 7 points below.
AE.



Set 4 - 13 points each

10. Compute the volume of the tetrahedron with vertices at $(0, 0, 0), (1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$.

11. Two circles ω_1 and ω_2 have the same center O. The area of ω_1 is greater than the area of ω_2. The radius of ω_2 is 2. Choose points A, B, C, D on a line in this order such that A and D lie on ω_1 and B and C lie on ω_2. If $AB = BC = CD = OB = OC$, find the area of ω_1.

12. A circle has diameter AB and area 4π. A second circle passes through A and B but has area 8π. Find the area inside both circles.

Set 5 - 14 points each

13. Jeffery the Jangaroo starts at his house and walks in a straight line 30 miles to the nearest bathroom. He then walks in a straight line 2017 miles from the bathroom to the nearest refrigerator. What is the minimum possibe distance between the bathroom and the refrigerator, in miles?

14. Jeffery, Jorge, a Jiraffe, and a Jeetah are standing in a line. Because Jeffery and Jorge are strange, they must each stand twice as far from the Jeetah than the Jiraffe. Because Jeffery and Jorge don't like each other, they stand at different points. Jeffery and Jorge are standing 18 meters apart. The Jiraffe and the Jeetah are standing k meters apart. Find k.

15. A circle with center (a, a) passes through the point $(4, -15)$. If a is the smaller root of $x^2+11x-2017 = 0$, find the area of the circle.

Set 6 - 15 points each

16. Jeffery is a sphere. Consider a points A and X such that AX is tangent to Jeffery at X. $AX = \sqrt{2017}$. Another line is drawn through X which intersects Jeffery at two points Y and Z such that $AY < AZ$ and AY and AZ have integer lengths. Find the sum of all distinct values of $AZ - AY$.

17. Let H denote the orthocenter of $\triangle ABC$. AH, BH, CH intersect the circumcircle of $\triangle ABC$ again at X, Y, Z respectively. If $\angle YXZ = 34°$, find $\angle BAC$.

18. In $\triangle ABC$, $\angle B$ is right and $AB = BC = 4$. Choose points D and E on AB and AC respectively such that $\angle BDC = \angle BEA = 60°$. Let F be the reflection of D across B. Find the length of EF.

Set 7 - Up to 20 points each

19. How many distinct triangle with integer side lengths are there with perimeter equal to 2017? If N is your answer and A is the correct answer, then your score will be calculated as $20 \min\left\{\frac{A}{N}, \frac{N}{A}\right\}$.

20. Jeffery the Jangaroo is feeding his massive troop of 20172017 kangaroos. However, he only has one huge circular pizza. What is the minimum number of straight cuts Jeffery has to make to divide the pizza into 2017 (not necessarily equal) pieces? If N is your answer and A is the correct answer, then your score will be calculated as $20 \min\left\{\frac{A}{N}, \frac{N}{A}\right\}$.

21. (Yes or No) The answer to each of these questions is either yes or no. You may answer each question either yes or no, or leave it blank. If you answer any of the questions incorrectly, then you will receive a 0 on this problem. Otherwise, if you answer n questions correctly, you will receive $(n-1)(n-2)$ points. Your answer should be a string of 6 letters Y for yes, N for no, 0 for blank. For example, your answer could be $Y0NYY0$.

1. Is the incircle of a triangle always smaller than its circumcircle?
2. Is the smallest circle containing 4 concyclic points always its circumcircle?
3. Is the orthocenter of a triangle always inside the triangle?
4. Pythagorean triples are triples of integers such that $a^2 + b^2 = c^2$. For example, $3^2 + 4^2 = 5^2$. Are there triples of positive integers where $a^3 + b^3 = c^3$?
5. Is it possible to use a ruler and compass to trisect any angle?
6. In $\triangle ABC$, let M, N, P be the midpoints and D, E, F be the feet of the altitudes. Are M, N, P, D, E, F concyclic?