1 Set 1 - 10 points each

1. A triangle's base is twice the length of its height. If its base has length 4, find the area of the triangle.

2. Square $ABCD$ has side length 4. Let the midpoints of BC and CD be M and N. Find the area of $\triangle AMN$.

3. In $\triangle ABC$, let the midpoints of dAB and AC be M and N. Denote the centroid of $\triangle ABC$ as G. If the area of $\triangle ABC$ is 24, find the area of quadrilateral $AMGN$.

2 Set 2 - 10 points each

4. A cube has surface area 48. Find its volume.

5. Equilateral $\triangle ABC$ is inscribed in circle ω. If $AB = 6$, find the area of ω.

6. In $\triangle ABC$, let D and E be the feet of the altitudes from B to AC and C to AB respectively. If $AD = 3, CD = 4$, and $AB = 6$, find the length of AE.

3 Set 3 - 10 points each

7. Jeffery the Jangaroo is still looking for his stuffed Jiraffe, but now he has developed a more complicated strategy. He walks 10 feet east, but does not find it. He continues and walks 30 feet north, but still does not find it. So he walks another 50 feet west. Unable to locate his beloved stuffed Jiraffe, he walks back to where he started and cries. How far did he walk in total?

8. Jeffery randomly chooses a integer $4 \leq x \leq 20$. $\triangle ABC$ has side lengths $5, 12$, and x. Find the number of values of x for which $\triangle ABC$ is acute.

9. Jeffery is a Jangaroo. In quadrilateral $ABCD$, the internal angle bisectors of $\angle A$ and $\angle C$ intersect on segment BD. If $AB = 110, BC = 140$, and $CD = 180$, find the maximum possible integer length of BD.

4 Set 4 - 15 points each

10. Right triangle ABC has side lengths $3, 4$, and x. Find the smallest possible value of x.

11. Rhombus $ABCD$ has diagonals AC and BD which intersect at E. Let the foot of the altitude from D to AB be P. Find DP if $AE = 4$ and $BE = 3$.

12. Let T be a point on circle ω with radius 6. Let A be a point in the plane such that AT is tangent to ω and $AT = 4$. Let B be the point such that TB is a diameter of ω. ω meets line segment AB at points B and C. Find $\frac{AC}{BC}$.

5 Set 5 - 15 points each

13. Kenny randomly chooses a point P inside a circle centered at O with radius 4. Find the probability that $OP \leq 2$.

14. Regular pentagon $ABCDE$ has side length 3. Let the foot of the altitude from A to $\overline{BE}$ be M and the foot of the altitude from A to $\overline{BD}$ be N. The length of MN can be written as $a \cdot \cos b$, where $0 \leq b \leq 180$ in degrees and a and b are both nonnegative integers. Compute ab.

15. Quadrilateral $ABCD$ is inscribed in circle ω with radius 5. If $AB = 6, BC = 7$, and $CD = 8$, find the length of DA.

6 Set 6 - 15 points each

16. Let O be the origin of the xy-plane. Let A and B be points on the parabola $y = x^2$ such that $\triangle ABO$ is equilateral. Find the area of $\triangle ABO$.

17. Triangle ABC has side lengths $AB = 3, BC = 4$, and $CA = 5$. Points W and X are chosen on segments AB and BC respectively. Let Y and Z be points on segment AC. If $WXYZ$ is a square, find its side length.

18. Given $x, y, z > 0$ satisfying

$$x^2 - xy + y^2 = z^2,$$
$$x^2 - xz\sqrt{3} + z^2 = y^2,$$

compute the maximum possible value of $\frac{y}{z}$.

7 Set 7 Estimation - 20 points each

If A is your answer and C is the correct answer, your score is calculated as $20 \cdot \min\left(\left(\frac{A}{C}\right)^2, \left(\frac{C}{A}\right)^2\right)$.

19. Kenny has a large hollow sphere with radius $10^{1,000,000}$ which he calls Jeffery. Kenny also has an infinite number of spheres with radius $10^{-1,000,000}$, each of which is called a Steve. Kenny optimally fills as many Steves as he can inside of Jeffery. Estimate the percentage of Jeffery's volume that is filled with Steves.

20. A, B, and C lie on a line in that order such that $BC = 10$. Let K be a point such that $AK \perp AB$ and $AK = 10$. If the inradius of $\triangle KBC$ has length 10^{-100}, estimate the length of AB in scientific notation.

21. Jonathan and Daniel are racing. At 1 second, Jonathan jumps forward 12 units and Daniel jumps forward 1 unit. At 2 seconds, Jonathan jumps backwards 6 units and Daniel jumps forwards $\frac{1}{2}$ units. At 3 seconds, Jonathan jumps forwards 3 units and Daniel jumps forwards $\frac{1}{3}$ units. They continue this where at the nth second, Jonathan jumps forwards $12(-0.5)^{n-1}$ units and Daniel jumps forwards $\frac{1}{n}$ units. Estimate how many seconds it takes Daniel to catch up to Jonathan.