Set 1 (10 points each)

1. Kenny walks 5 feet north and 12 inches south. How many feet is he from where he started?

2. $\triangle ABC$ is iscosceles. Given that $\angle A = 30^\circ$, find the maximum value of $\angle B$.

3. $\triangle ABC$ is iscosceles with $AB = AC$. Let D be a point on AC such that $BD \perp AC$. If $BD = 3$ and $CA = 5$, find BC.

Set 2 (11 points each)

4. The volume of a cube is 4 times its side length. Find its surface area.

5. The interior angles of a regular polygon measure 170°. Find the number of sides of this polygon.

6. Consider a triangle with side lengths $3, 4$, and 5. Find OH where O is the circumcenter and H is the orthocenter.

Set 3 (12 points each)

7. In $\triangle ABC$, $AB = 3, BC = 4$, and $CA = 5$. Square $ABXY$ is drawn outside $\triangle ABC$. Find the area of $\triangle AXC$.

8. Daniel wants to place two circular cookies, of radius r and $2r$ in his square box of side length $2 + \sqrt{2}$. Given that his cookies cannot overlap, find the maximum possible value of r.

9. Isosceles triangle ABC has $AB = AC = 10$ and $BC = 16$. A point G is constructed as shown below such that G is the centroid of $\triangle ABC$. Segment GD is drawn such that GD is perpendicular to AC. Find AD.

Set 4 (13 points each)

10. In $\triangle ABC$, $AB = 3, BC = 4$, and $CA = 5$. Let G be its centroid. Find AG.

11. Points A, B, C, and D are chosen on a circle such that AB and CD intersect at a point P inside the circle. Given that $AP = 2$, $BP = 6$, and CP and DP have integer lengths. Find the number of possible values of CP.

12. Given a square $ABCD$ and any point P such that $[ABP] = 10, [BCP] = 14$, and $[DAP] = 18$, find the maximum possible value of $[CDP]$. Note that $[XYZ]$ denotes the area of $\triangle XYZ$.

Set 5 (14 points each)

13. A quadrilateral $ABCD$ satisfies $AB = 10, BC = 12, CD = 14$, and $DA = 20$. Find the greatest possible length of one of its diagonals.

14. Let ABC be a triangle with $\angle BAC = 60^\circ$, $AB = 15$, and $AC = 20$. Let B' be the reflection of B over AC and C' be the reflection of C over AB. Compute the area of $AB'C'$.

15. Consider $\triangle ABC$, where $AB = 10, BC = 11$, and $CA = 12$. Three points X, Y, and Z are chosen inside $\triangle ABC$. Find the probability that $AX \geq BY \geq CZ$.

Set 6 (15 points each)

16. A circle ω is inscribed in a square $ABCD$. Let BD intersect ω at two points P and Q. Let ω be tangent to AB at X. If $AB = 4$, find the area of $\triangle PQX$.

17. Let ABC be a triangle with $AB = 3$, $BC = 7$, and $AC = 8$. Let D, E, F be the midpoints of AB, BC, and AC, respectively. Suppose the altitudes of DEF intersect at a point H. Compute $\max(HA, HB, HC) - \min(HA, HB, HC)$. (For real numbers a, b, c, $\max(a, b, c)$ denotes the largest value out of the 3, and $\min(a, b, c)$ denotes the smallest value out of the 3.)

18. Circle α and β are internally tangent at point C with circle α bigger than circle β. The length of the part of diameter CD outside circle β is 6. Diameter EF is perpendicular to CD. The length of the part of EF outside circle β is 8. Find the radius of circle α.

Set 7 (20 points each)

19. Let N be the number of lattice points contained in the sphere $x^2 + y^2 + z^2 \leq 2017$. Calculate N. (Note that a lattice point is a point with integer coordinates. For example, $(-1, 2, 5)$ is a lattice point while $(\pi, 3, \sqrt{2})$ is not.

20. Let $O = (0, 0)$, $P = (10^{100}, 0)$, and ℓ_0 be the line $y = x$. For $n \geq 1$, define ℓ_n to be the line that bisects the acute angle between ℓ_{n-1} and the x-axis. Determine the smallest n such that there exists a point X on ℓ_n such that $PX \leq 1$.

21. (True/False) The answer to each of these questions is either yes or no. You may answer each question either yes or no, or leave it blank. If you answer any of the questions incorrectly, then you will receive a 0 on this problem. Otherwise, if you answer n questions correctly, you will receive $(n-1)(n-2)$ points. **Circle** the numbers of statements you think are true and **cross out** the numbers of statements you think are false. Do not mark numbers of statements which you are unsure of.

1. The circumcenter of a triangle is always inside the triangle.
2. There exists a triangle whose perimeter is 10,000 times its area.
3. Let O be the circumcenter of $\triangle ABC$. Then the inequality $OA + OB + OC \leq AB + BC + CA$ always holds.
4. The circumcenter, orthocenter, and centroid of a triangle are always collinear.
5. Given $\triangle ABC$ and a point P, let the concurrence point of the lines AP, BP, and CP across the angle bisectors of $\angle A$, $\angle B$, and $\angle C$ respectively is called the isogonal conjugate of P. Is the following statement true: **the circumcenter and orthocenter are isogonal conjugates.**
6. The incenter is the only point that is always the isogonal conjugate of itself.

Guts Answers

Set 1 1. 4 2. 120 3. $\sqrt{10}$

Set 2 4. 24 5. 36 6. 5/2

Set 3 7. 21/2 8. 2/3 9. 12/5

Set 4 10. $\frac{2\sqrt{13}}{3}$ 11. 6 12. 42

Set 5 13. 26 14. 0 15. 8

Set 6 16. $2\sqrt{2}$ 17. 0 18. 8

Set 7 19. 379149 20. 333 21. XOXOOX