

## 2010 Amador Valley Geometry Bee

May 19<sup>th</sup>, 2010 @ AVHS  
Sponsored by the AV Math Team and  
Math Department

### Round 1

- 1 minute per problem
- 10 problems total
- Calculators OK!
- Good luck!

### Question 1-1

- The radius of a circle is decreased by 20%. By what percent is its area decreased?

### Question 1-2

- The measures of the interior angles of a particular triangle are in a 5:6:7 ratio. What is the measure, in degrees, of the smallest interior angle?

### Question 1-3

- The area of a circle is  $49\pi$  square units. What is its radius?

### Question 1-4

- Two rectangles have integer dimensions, and both have a perimeter of 100. What is the greatest possible difference between the areas of two such rectangles?

### Question 1-5

- How many axes of symmetry does a regular pentagon have?

### Question 1-6

- What integer is closest to the area of a triangle whose sides are 5, 6, and 7 units?

### Question 1-7

- If the lengths of the three sides of a right triangle are whole numbers, how many such distinct right triangles exist having one leg with a length of 8 units?

### Question 1-8

- A rectangular prism with whole number dimensions has faces with areas 54, 72, 108 square inches. How many cubic inches are there in its volume?

### Question 1-9

- Alice and Bob are standing 20 feet apart. Each of them throws a rock at the other at the same time, and the rocks follow a perfectly semicircular path (with diameter 20). Bob throws his rock three times faster than Alice throws hers. The rocks collide at point P. If the distance from P to Alice is  $x$ , find  $x^2$ .

### Question 1-10

- Who wrote the math book *Elements*?



## Round 2

- ◉ 2 minutes per problem
- ◉ 8 problems total
- ◉ Calculators OK!
- ◉ Good luck!

## Question 2-1

- ◉ An equilateral triangle of side length  $t$  has area equal to that of a regular hexagon of side length  $h$ . Find  $(t/h)^2$ .

## Question 2-2

- ◉ The sum of all but one of the interior angles of a convex polygon equals 2570. What is the remaining angle?

## Question 2-3

- ◉ A man walks  $x$  miles on a flat plane due west, turns 150 degrees to his left and walks 3 miles in the new direction. If he finishes at a point  $\sqrt{3}$  from his starting location, what are all possible values of  $x$ ?

## Question 2-4

- ◉ A vertical line divides the triangle with vertices  $(0, 0)$ ,  $(1, 1)$ , and  $(9, 1)$  in the  $xy$ -plane into two regions of equal area. The equation of the line is  $x = ?$

## Question 2-5

- ◉ An isosceles triangle has side lengths  $3x + 4$ ,  $4x - 3$ , and  $7x$ . What are all possible values of  $x$ ?

### Question 2-6

- What is the length of the shortest altitude of a 3-4-5 triangle?

### Question 2-7

- The corner of a unit cube is chopped off and discarded such that the cut runs through the three vertices adjacent to the vertex of the chosen corner. What is the height of the cube when the freshly-cut face is placed on a table?
  - > (question from HMMT)

### Question 2-8

- A polyhedron has faces that are all either triangles or squares. No two square faces share an edge, and no two triangular faces share an edge. What is the ratio of the number of triangular faces to the number of square faces?
  - > (question from HMMT)

### Round 3

- 3 minutes per problem
- 6 problems total
- NO CALCULATORS
- Good luck!

### Question 3-1

- Let  $T$  be a right triangle with sides having lengths 3, 4, and 5. A point  $P$  is called epic if  $P$  is the center of a parallelogram whose vertices all lie on the boundary of  $T$ . What is the area of the set of epic points?
  - > (question from HMMT)

### Question 3-2

- In  $\triangle ABC$ , points  $D$ ,  $E$ , and  $F$  lie on  $\overline{BC}$ ,  $\overline{AC}$ , and  $\overline{AB}$ , respectively, such that  $3 \cdot CD = BC$ ,  $3 \cdot AE = AC$ , and  $3 \cdot AF = AB$ . The pairwise intersections of  $\overline{AD}$ ,  $\overline{BE}$ , and  $\overline{CF}$  are  $P$ ,  $Q$ , and  $R$ . If the area of  $\triangle ABC$  is 84, then what is the area of  $\triangle PQR$ ?

## Question 3-3

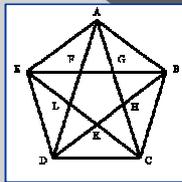
3. The side lengths of a parallelogram are 3 and 8. Given that its angle measures are  $15^\circ$  and  $165^\circ$ , what is the sum of the squares of the lengths of its diagonals?

## Question 3-4

4. In an  $8 \times 10$  grid of squares, how many squares exist such that their vertices lie on the grid and their edges are parallel to the sides of the grid?

## Question 3-5

- ⦿ In regular 5-pointed star ABCDEFGHKL (shown), what is the ratio of the area of pentagon ABCDE to that of FGHKL?



## Question 3-6

6. A circle and 100 lines are drawn in the plane. If the maximum number of regions the circle may be divided into is  $A$  and the minimum is  $B$ , then what is  $A + B$ ?