Amador Valley Geometry Bee

May 23, 2011

Format

- Round 1: 10 problems, 2 minutes per problem
- Round 2: 8 problems, 2 minutes per problem
- Tiebreakers will be used to determine placements

Rules

- Write down your answers on the paper given.
- Answers **must** be in simplest and exact form.
- NO Calculators
- Good Luck!

Round 1

10 problems, 2 minutes per problem

Problem 1

What is the side length of an equilateral triangle whose perimeter (in units) and area (in units^2), have the same value?

Problem 2

What is the sum of all possible integral side lengths of a side of a triangle whose other sides are 20 and 11?

Problem 3

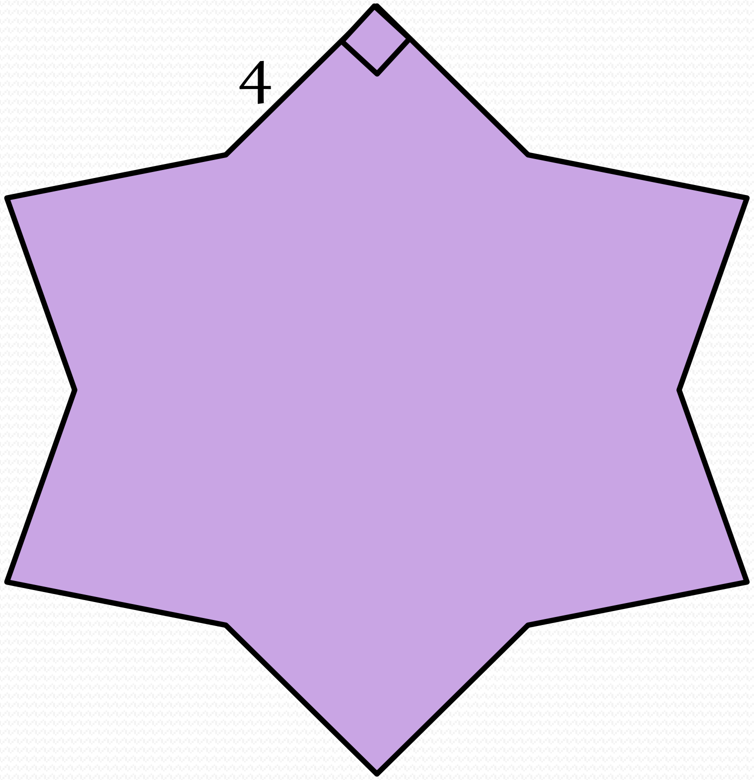
What is the area of the region bound by $y = \sqrt{324 - x^2}$ and the x-axis?

Problem 4

Jack has declared war on his neighbor, Jill. From his 30' vantage point in the tree, he can barely see the top of Jill's doghouse over the 12' fence. He has measured the distance from the base of the tree to the fence to be 60'. Jack has also clocked his 40' army crawl to be 30 seconds. If the doghouse is 4' tall, how long, in seconds, should Jack project for it to take him to crawl to the doghouse once he digs under the fence?

Problem 5

What is the area of a equilateral 6-pointed star with side length 4 if the points are right angles?

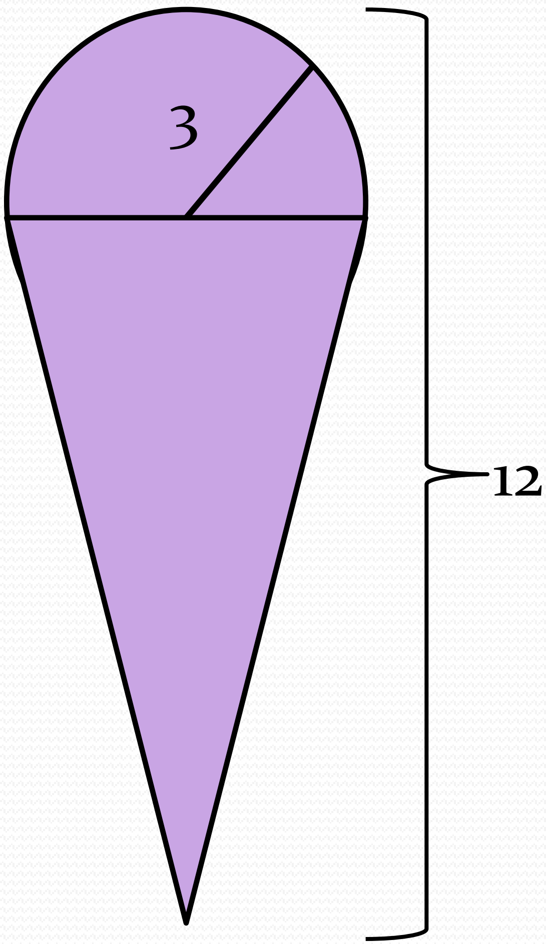


Problem 6

What the maximum area of a rectangle inscribed in a semicircle of radius 12?

Problem 7

Jill is eating an ice cream cone composed of a cone with a hemisphere on top. Both radii are 3cm, and the height of the entire treat is 12cm. If Jill eats π cm^3 every second, how long will it take, in seconds, for the ice cream to reach one-fourth of its original height?



Problem 8

Three points on a circle are chosen randomly and independently. What is the probability that the points are the vertices of an acute triangle?

Problem 9

In $\triangle ABC$, $\angle ACB$ is a right angle. $AB = 13$, $BC = 12$, and $AC = 5$. The smallest circle through C that is tangent to segment AB intersects segment AC at J and segment BC at K. Find JK.

Problem 10

Who is considered "The Father of Geometry?"

Round 1 Tiebreaker

2 minutes per problem

Problem 1

While demonstrating Archimedes' Principle, a student fills a tall cylindrical beaker of radius 8 cm to the brim with water and places it in a flat cylindrical beaker of radius 10 cm. He then lets a ball of radius 6 sink into the water-filled beaker so that water overflows. What is the height, in cm, of the water level of the outside beaker?

Answer: 8

Problem 2

A dartboard consists of 12 concentric circles with radii 1, 2, 3, …, 12. The regions are colored in alternating red and black, with the bulls-eye colored red. If a dart lands randomly on the board, what is the probability it lands on a red area?

Answer: $\frac{11}{24}$

Round 2

8 problems, 3 minutes per problem

Problem 1

Stan the starman notices that he can fly through xyz space 1110 kilometers in the x direction, 1210 kilometers in the y direction, and 1161 kilometers in the z direction to get from his home star to his friend's planet. If he reroutes his course to attain the minimum distance, what is the length, in kilometers, of the new path?

Problem 2

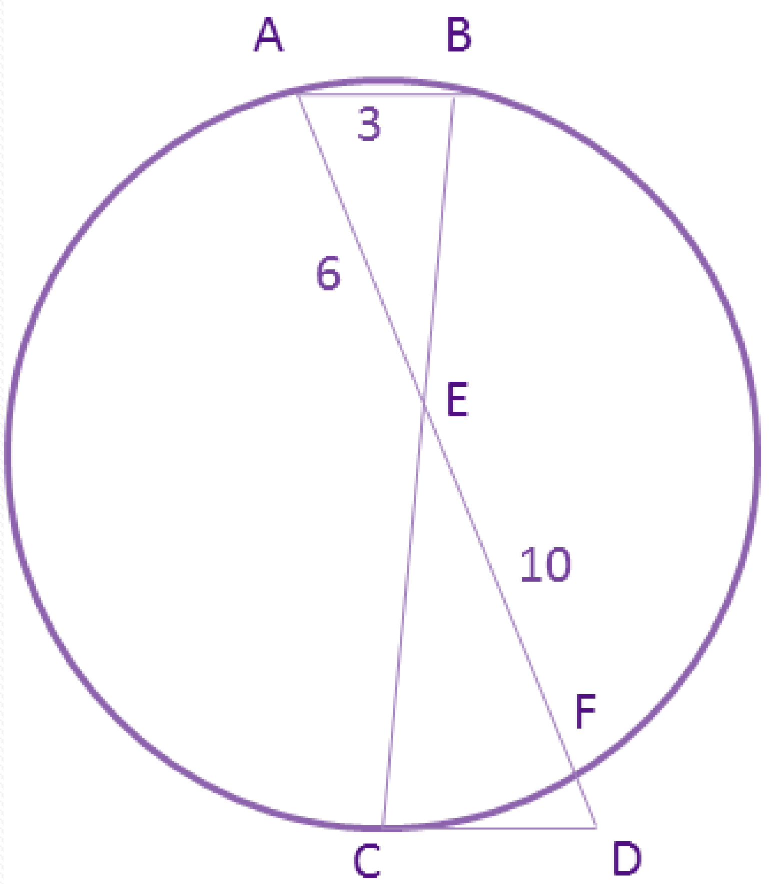
In $\triangle ABC$ let $AB = 15$, $AC = 13$, and D be the foot of the altitude from A to BC. $AD = 12$. If E is on AB such that $BE = CE$, then find the measure of segment BE.

Problem 3

Team Rocket has imprisoned Pikachu in an 8’ cube cage and placed Pikachu on electricity-resistant ground in the middle of an open field. If Pikachu’s thunderbolts have a range of 12’ and Pikachu can navigate to all points within the cage, what is the volume, in $feet^3$, of Pikachu’s attack range outside the cage?

Problem 4

Let A, B, C, and F be points on a circle. Arc AC is congruent to arc BC. Let E be the intersection of AF and BC, and let D be the intersection of AF and the tangent to C. If AB=3, AE=6, and EF=10, then what is DF?



Problem 5

A regular tetrahedron has a volume of 6. Additional regular tetrahedra are annexed onto each of the faces such that three of the vertices of the attached tetrahedra lie on the midpoints of the edges of the old figure.
If this process is repeated indefinitely, what is the volume of the infinite shape?

Problem 6

What is the sum of all possible heights of a right circular cone given that it has surface area 96π and volume 96π?

Problem 7

What is the volume of the shape in the xyz coordinate system bounded by $x^2 + y^2 < 9$ and $x^2 + y^2 > z^2$?

Problem 8

An empty circular conic cup is filling at a rate of 3π cm^3/s. If the cup has a radius of 4 cm and a height of 12 cm, how fast, in cm/s, is the water level rising at 9 seconds?

Tiebreaker: Countdown Round

Rules

- Point awarded to first correct answer.
- 90 second time limit.
- Best out of three.

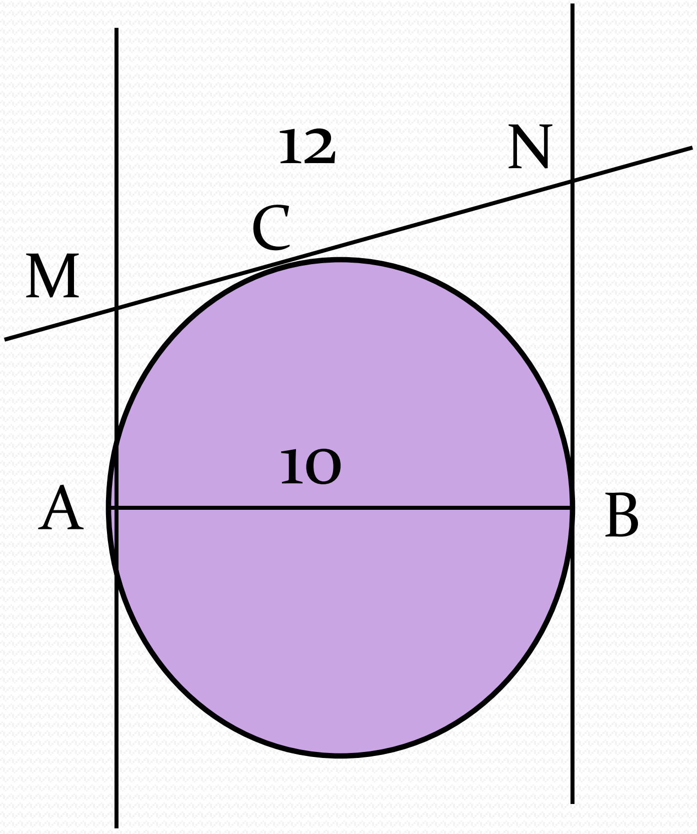
Problem 1

What is the positive difference between the maximum and minimum number of regions that 2011 distinct lines can split an infinite plane into?

Answer: 2021055

Problem 2

Point C lies on a circle with diameter AB such that the tangent to C intersects the tangents of A and B at M and N, respectively. If the circle has diameter 10, and MN=12, what is the area of quadrilateral AMNB?



Answer: 60

Problem 3

A scalene triangle has integral side lengths. If the perimeter is 42, what is the maximum area?

Answer: 84

Problem 4

What is the volume of a regular tetrahedron with side length 6?

Answer: $18\sqrt{2}$

Problem 5

A square and a circle have the same center and both have an area of π. What is the total length of the portions of the square's perimeter that are inside the circle?

Answer: $4\sqrt{4-\pi}$

Problem 6

Wheel A, with diameter 4, rolls around Wheel B, with diameter 60, without slipping. How many complete revolutions does Wheel A make as it makes one full revolution around Wheel B?

Answer: 16

Problem 7

What is the least possible area of a quadrilateral circumscribed about a semicircle of radius 3?

Answer: $9\sqrt{3}$

Problem 8

Suppose we have a piece of paper in the shape of an equilateral triangle. Define a midpoint cut as cutting the paper through the midpoints of two sides, forming two pieces. One is discarded, and the other piece is kept to make another cut. What is the minimum number of midpoint cuts necessary to obtain a rectangle?

Answer: 4

Problem 9

In triangle ABC, $\sin A = \cos B = \cos C$. If its circumcircle has radius 2, what is the perimeter of the triangle?

Answer: $4 + 2\sqrt{3}$

Problem 10

A regular tetrahedron has a greater surface area (in units^2) than volume (in units^3). What is the sum of all possible integral side lengths?

Answer: 105

Thank you for Coming!