Amador Valley Geometry Bee
Qualifying Written Round SOLUTIONS
30 minutes

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1. **2308 kilometers (+/- 1 for rounding errors).** 423,970 is the new area of our circular state. Using the equation 3.14r^2, we divide the area by pi, take the square root, and get the radius of our state. The circumference is 2*pi*r, we multiply the radius by 3.14 and 2 and get 2308, rounded to the nearest kilometer.

2. **49 degrees.** Given angles A, B, and C, we know that 2B = A and B-16 = C. Angles A, B, and C will add up to 180. A + B + C = 180. Substituting for A and C in terms of B, we get the equation 2B + B + B - 16 = 180. Solving for B, the measure of angle B is 49 degrees.

3. **35 meters**.There are similar triangles. The height of the flagpole is to the height of its shadow as the height of the tree is to the height of its shadow. 15/9 = height/shadow. Shadow = height - 14. 15/9 = h/(h-14). Cross multiply and solve for h. The height of the tree is 35 meters.

4. **60/13 or 4.62 or 4.615**. The shortest altitude connects with the side of length 13. Denote this altitude h. Notice that (13h)/2 defines the area of the triangle in the same way that (12*5)/2 does. We have (13*h)/2 = (12*5)/2. Solving for h, we have that h = 60/13 or 4.62.

5. **41.** The side lengths of the triangle are x, 4x, and 16. By the triangle inequality theorem, x + 4x > 16, 4x + 16 > x, and 16 + x > 4x. Simplifying these inequalities, we get 5x > 16, 3x > -16, and 16 > 3x. The maximum value of x that satisfies all three of those inequalities is 5. The side lengths are 5, 20, and 16. The perimeter is 41.

6. **126 x 126 meters.** Draw a picture and find similar triangles. Let us define s as half of a side length of the fort. There are two key triangles, one with side lengths 42 and s and the other one with side lengths 42 + 2s + 42 and 315. These triangles are similar, so their sides must be proportional. We have the proportion 42/2s = (84 + 2s)/315. Simplifying, we get 2s^2 + 84s - 13230 = 0. Solving for s, we get s = 63 or -105. s denotes half a side length, so the full dimensions of the fort are 126 x 126.

7. **104 (+/- .1 for radical errors).** Draw a diagram with the coordinates mentioned, and circumscribe the triangle within a box such that the triangle is now bounded by three more triangles. The area of the triangle is equal to the area of a square minus the area of the three triangles around the main triangle. [20*12] - [4*20/2 + 8*12/2 + 8*12/2] = 240 - 136 = 104.

8. **27sqrt(3).** Notice that if one of the points must be the origin, the next two points must have the same y value, and the two points must form a segment that's parallel to the x-axis. Given that, you see that the triangle must be "upside down" balancing on a point on the origin. Using trigonometry, you can determine that the slope of the segment of the triangle in the first quadrant is tan(60), or sqrt(3). Given that the segment passes through the origin and has a slope of sqrt(3), you know that one of the points must lie on the line y=sqrt(3)x. Now, using systems of equations with the two lines, y=sqrt(3)x and y=(x^2)/3, you can solve for one point, (3sqrt(3), 9). The other point is a reflection of that point on the y axis, and becomes (-3sqrt(3), 9). This is an equilateral triangle of height 9 and base 6sqrt(30). The area is 27*sqrt(3).

9. **122.8.** Notice that the needed area is made out of two equilateral triangle and four (moon-shaped) circle segments. The base of equilateral triangle has a length of 10, so the area of the triangle is 25sqrt(3). The area of each circle segment is equal to (circle slice of angle 60) - (equilateral triangle) = (100pi/6 - 25sqrt(3)). Adding together the areas of two triangles and four segments gets 2[25sqrt(3)] + 4[100pi/6 - 25sqrt(3)] = 122.8

10. **29 (+/- 1 for rounding errors).** Angle O is half of the measure of arc MN. The measure of angle O is 75, the measure of MO is 2n, and the measure of ON is n. Using the law of cosines, we get that MN^2 = MO^2 + NO^2 - 2(MO)(NO)cos(O). Substituting, we get MN^2 = 4n^2 + n^2 - 4n^2cos(58). Simplifying, we get MN = n*sqrt(5-4cos(58). Now, to find angle M, we use the law of sines. <M = ?, <O = 58, NO = n, MN = n*sqrt(5-4cos(58). sin(M)/n = sin(58)/(n*sqrt(5-4cos(58)). The n is just a ratio, it cancels out. Simplifying, we get sin(M) = sin(58)/sqrt(5-4cos(58). Solving, we get <M = 29.

11. **30%.** Any three points must lie in the same plane. Thus, the centers of the circles all lie in the same plane inside the larger circle. We can now simplify this to a circle problem, find the radius of three congruent externally tangent circles inside of a larger circle with radius 20. Draw a diagram of such, and label the radius of the larger circle. Connect the centers of the circle to make an equilateral triangle. Using properties of a 30-60-90 triangle and solving for r, the radius of one of the smaller circles, you can find that r + 2r/sqrt(3) = 20. Solving for r, you find that r = 9.282. Now, to find the percentage volume occupied, return to the spheres and calculate then divide the relative volumes. 3[4/3(pi)r^3] / [4/3(pi)R^3] = 3[4/3(pi)9.282^3] / [4/3(pi)20^3] = .300 = 30%