Amador Valley's 6th Annual Geometry Bee

Invitational Round

May 23, 2013

Format

- Round 1: 10 problems, 2 minutes per problem
- Round 2: 10 problems, 2 minutes per problem
- Round 3: 10 problems, 3 minutes per problem
- Tiebreakers will be used to determine placements, if necessary
- Top 16 highest scores on Round 1 will advance to Round 2, top 8 highest scores on Round 2 will advance to Round 3
- All students that don't advance above compete in the "Division 2" bracket, where they may win some other prizes, different from those that advance

Rules

- Write down your answers on the paper given.
- Unless otherwise stated, answers **must** be in simplest and exact form. Denominators must be rationalized, trigonometric functions must be simplified, common fractions must be used, etc.
- **No calculators!**
- Unless otherwise stated, assume angle measures are written in degrees.
- Good Luck!

Incorrect	$1\frac{1}{2} = 1.5$	$\cos(30^\circ)$	$\dfrac{1+\sqrt{5}}{6-2\sqrt{5}}$	$\sqrt{\dfrac{5}{2}}$	3^5
Correct	$\dfrac{3}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{2+\sqrt{5}}{2} = 1 + \dfrac{\sqrt{5}}{2}$	$\dfrac{\sqrt{10}}{2}$	243

Round 1

10 problems, 2 minutes per problem

1. If $ABCD$ is a rectangle with area 24 and integer side lengths, find the difference between the maximum perimeter and the minimum perimeter.

Start Pause Stop

Answer

2. In triangle ABC, D is on AC such that $\angle ABD = \angle CBD$. Given that $AB = 7$, $BC = 8$, and $AC = 5$, compute $\frac{AC}{AD}$.

Start Pause Stop

Answer

3. In semicircle with center O, AB is a diameter, and K is on the circumference of the semicircle. Compute $AK^2 + BK^2$, given that $KO = 7$.

Start Pause Stop

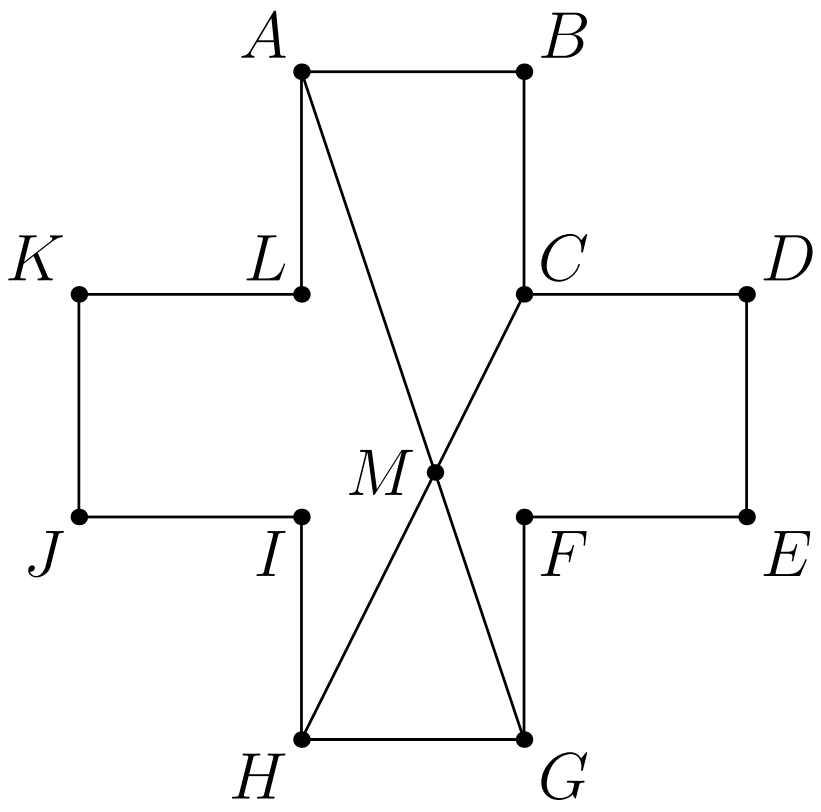
Answer

4. In triangle ABC, given that $AB = 3$, $BC = 4$, and $AC = 5$, and I as the incenter, compute $IA^2 + IB^2 + IC^2$.

Start Pause Stop

Answer

5. Consider the 12-sided polygon $ABCDEFGHIJKL$, as shown. Each of its sides has length 8, and each two consecutive sides form a right angle. Suppose that AG and CH meet at M. What is the area of quadrilateral $ABCM$?



Start Pause Stop

Answer

6. Three mutually tangent spheres of radius 1 rest on a table. A sphere of radius 3 rests on top of them. What is the distance from the top of the larger sphere to the table?

Start Pause Stop

Answer

7. A sphere is inscribed inside a cube. Given the cube's side length as 6, compute the volume inside of the cube but outside of the sphere.

Start Pause Stop

Answer

8. Find the area of a regular hexagon inscribed in a circle inscribed in a regular triangle if the triangle has side length 4.

Start Pause Stop

Answer

9. Brett, a dog with a poor sense of direction, begins 10 meters south of his doghouse. Suppose Brett begins to trot in a random direction until he is no longer moving towards the doghouse, that is, taking another step forward will increase the distance between Brett and his house. (Note that this could be right away: for example, if he begins by heading south.) At this point, if Brett is not home, he gives up and goes to sleep. What is the area, in square meters, of the region Brett might travel upon?

Start Pause Stop

Answer

10. Which of the following is a Pythagorean Triple? List the correct letter.

A) (100361, 134679, 167961)
B) (53672, 323830, 328248)
C) (26128, 160254, 162370)
D) (26836, 161915, 164124)

No calculators! :)

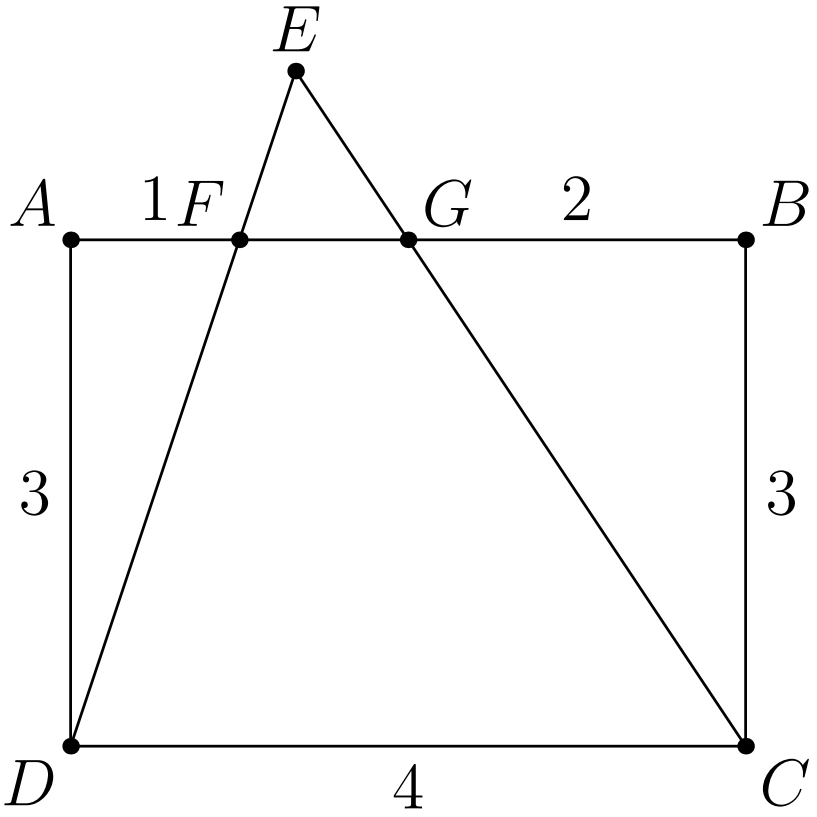
Start Pause Stop

Answer

Round 2

10 problems, 2 minutes per problem

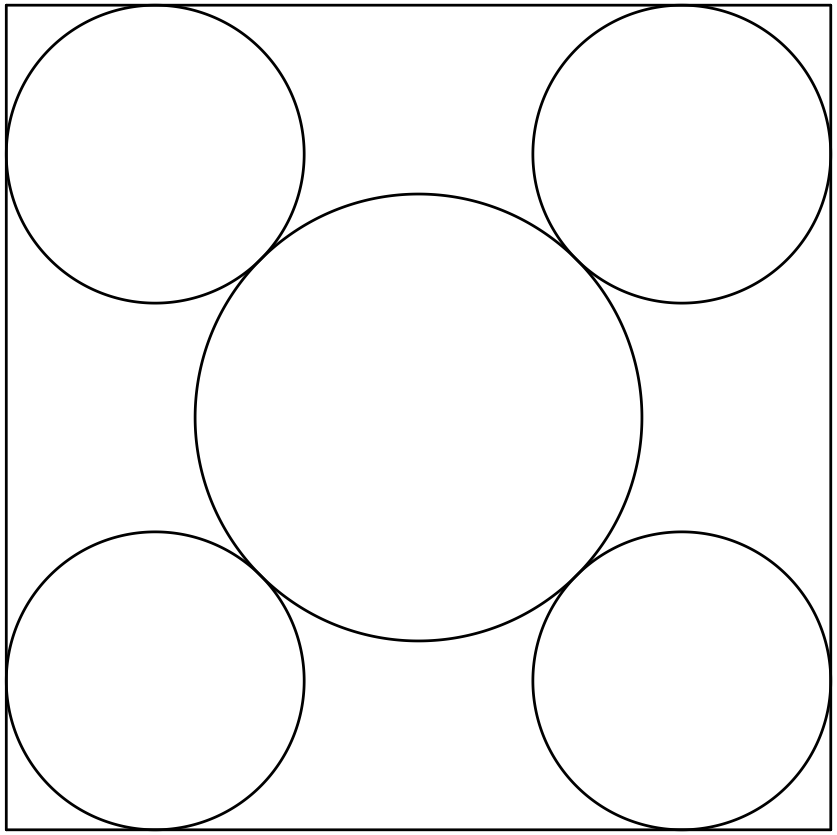
1. In rectangle $ABCD$, $DC = 4$ and $BC = 3$. Points F and G are on AB with $AF = 1$ and $BG = 2$. What is the perimeter of triangle DEC?



Start Pause Stop

Answer

2. Four circles with radii 2 are each tangent to two sides of a square and externally tangent to a circle of radius 3, as shown. What is the area of the square?



Start Pause Stop

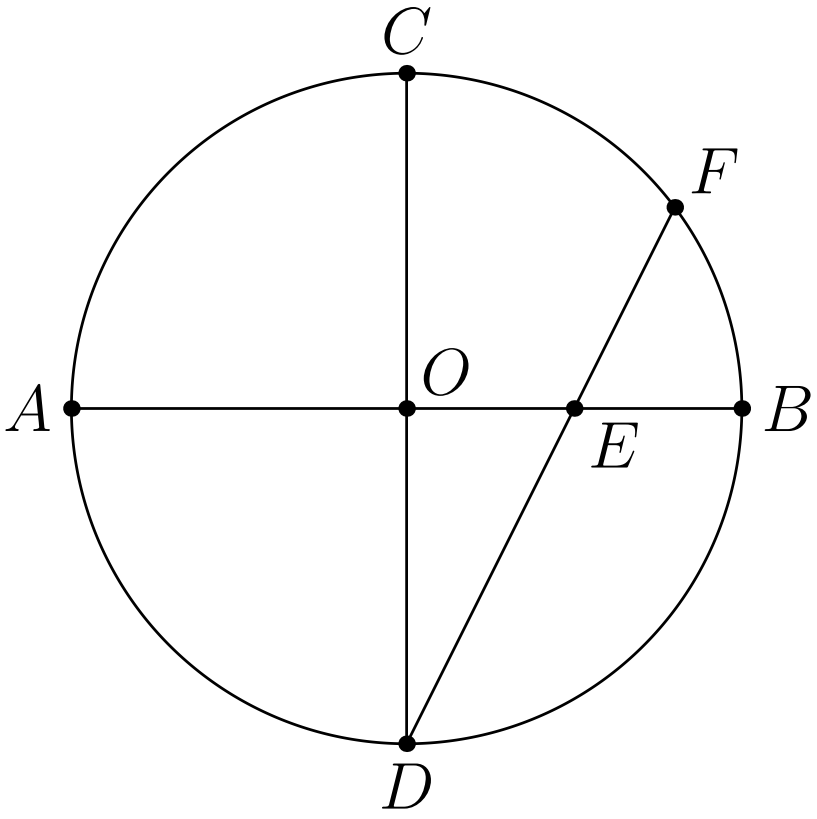
Answer

3. Equilateral triangle ABC has a side length of 6, and it is circumscribed by circle O. D is a point on minor arc BC such that $\angle BAD = \angle CAD$. Compute BD.

Start Pause Stop

Answer

4. In the figure, AB and CD are two perpendicular diameters of circle O. Chord DF intersects AB at E with $DE = 5$ and $EF = 3$. What is the area of circle O?



Start Pause Stop

Answer

5. Consider a circle, with point A randomly placed on the circumference. Point B is placed randomly and independently of A, also on the circumference. Given we place point C on minor arc AB, what is the probability that C is placed so that arc AC is more than $\frac{1}{6}$ of the total circumference of the circle?

Start Pause Stop

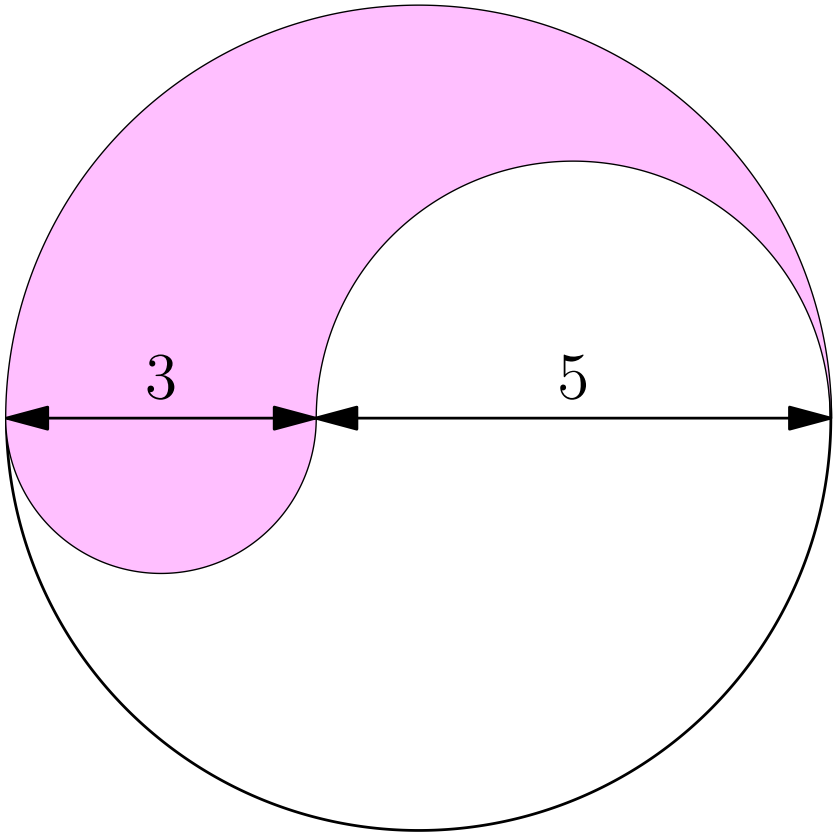
Answer

6. In triangle ABC and DEF, $DE = 3AB$, $DF = 3AC$, and $EF = 3BC$. Given that the area of DEF is 120 more than ABC, compute the area of DEF.

Start Pause Stop

Answer

7. The figure shown is the union of a circle and semicircles of diameters 3 and 5, all of whose centers are collinear. What is the ratio of the area of the shaded region to that of the unshaded region?



Start Pause Stop

Answer

8. Let $ABCD$ be a rectangle, and let E be on AB such that $2AE = EB$ and let F be on CD such that $CF = FD$. Let the intersection of AF and DE be P. Compute the fraction of the area of $ABCD$ that $BCFPE$ occupies.

Start Pause Stop

Answer

9. Three spheres are externally tangent to each other and tangent to a common plane. If the spheres have radii 4, 9, and 16, what is the perimeter of the triangle with vertices at the intersection of each sphere with the plane?

Start Pause Stop

Answer

10. In the 2D world of Flatland on the XY plane, there is a single point light source and the ground is defined by $y = 0$. A 6 foot pole on $x = 0$ casts an 8 foot shadow left and an 8 foot pole on $x = 10$ casts a 6 foot shadow left. Find the height of the light source.

Start Pause Stop

Answer

Round 2 Tiebreakers

You will have 3 minutes to solve one problem. When you solve a problem, raise your hand, and your proctor will grade it as well as assign you a time if you are correct. The participant with the lowest time moves on to the next round. In the case of both participants getting the problem wrong, we'll move on to a second problem, and so forth.

Go to Round 3

1. Ben is in a circular pen. If he goes 6 feet east, 4 feet north, or 8 feet west, he will encounter an electric fence and get shocked. What is the area, in square feet, of Ben's pen?

Start Pause Stop

Answer

Go to Round 3

2. A circle has radius 2 and is circumscribed about equilateral triangle ABC. If X is the midpoint of AC and Y is on arc AC such that $\angle YXA$ is right, then find XY.

Start Pause Stop

Answer

Go to Round 3

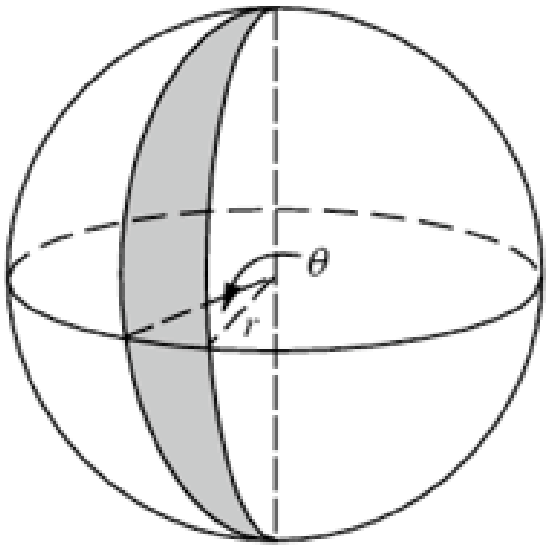
3. Monster Pac-Man is a growing sector of a circle. A baby Pac-Man starts with a radius of 0.25 cm and mouth opening of 11.25°. It doubles its radius and mouth opening each time it gobbles up a prey and continues growing until it reaches its adult size, which has a radius of at least 1.5 cm. When the baby Pac-Man stops growing (reaches adult size), what is its perimeter in cm?

Start Pause Stop

Answer

Go to Round 3

4. Pac-Man has a 3-D cousin Pac-Ball, which is a sphere with a small wedge removed, as shown in the figure. If the radius r is 1 cm, and the opening angle θ is 36°, what is the Pac-Ball's surface-area-to-volume ratio?



Start Pause Stop

Answer

Go to Round 3

5. What is the area of a triangle with side lengths 7, 8, and 9?

Start Pause Stop

Answer

Go to Round 3

6. Find the area of a rectangle which has a perimeter of 30 and its length is 3 more than its width.

Start Pause Stop

Answer

Go to Round 3

Round 3

10 problems, 3 minutes per problem

1. Akshay hates Rick, so he wants to cut up Rick's homework, which is a regular sheet of paper. However, Rick infused the paper with magic, so that the paper will only separate after seven complete cuts have been made. Assuming that he cannot bend the paper, what is the maximum number of pieces Akshay can cut Rick's homework into?

Start Pause Stop

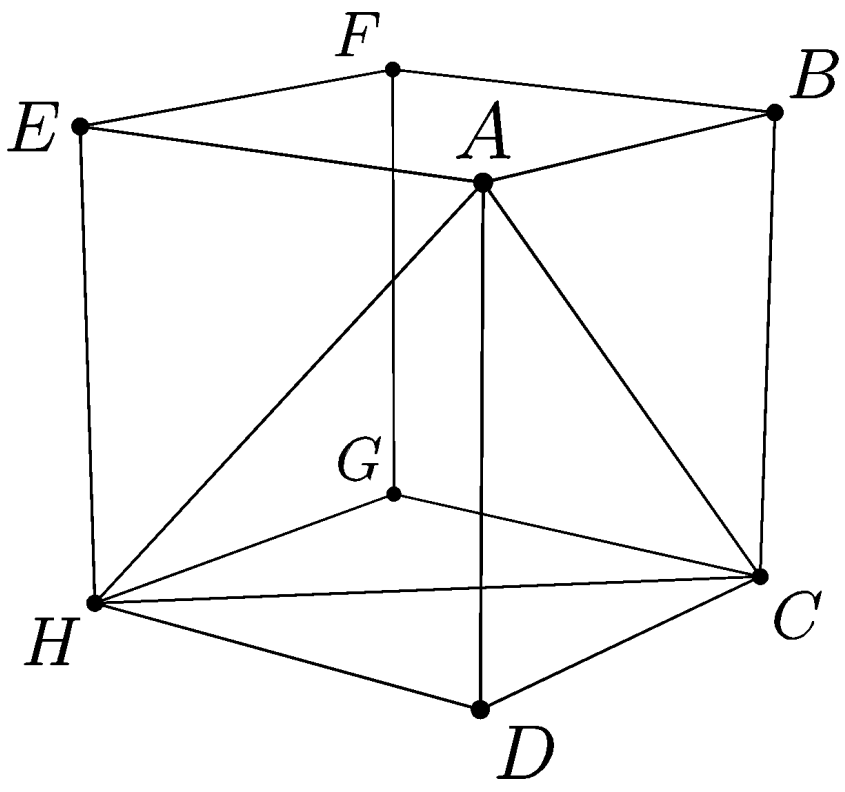
Answer

2. Let ABC be a triangle, and let the angle bisector of $\angle BAC$ intersect the circumcircle of ABC at D. The perpendicular from D to BC intersects BC at E. Given that $AB = 10$, $AC = 9$, and $BC = 8$, find BE.

Start Pause Stop

Answer

3. In a unit cube with side length 1 unit, vertices A, C, and H are adjacent to vertex D. Consider the plane containing the points A, C, and H. What is the shortest distance from D to this plane?



Start Pause Stop

Answer

4. Equilateral hexagon A_1 has an area of $6\sqrt{3}$. If A_2 is constructed by connecting the midpoints of the sides of A_1, and all A_n are constructed similarly until an infinite number of hexagons are constructed, compute the sum of the areas enclosed by the hexagons.

Start Pause Stop

Answer

5. Cyclic quadrilateral $ABCD$ has side lengths $AB = \sqrt{48}$, $BC = \sqrt{49}$, $CD = \sqrt{51}$, and $DA = \sqrt{52}$. Find the area of the circumcircle of $ABCD$.

Start Pause Stop

Answer

6. In triangle ABC, points D and E lie on BC and AB respectively, so that $BD = 2DC$ and $BE = 2AE$. AD and CE intersect at F. The area of triangle ABC is 24. Find the area of quadrilateral $FEBD$.

Start Pause Stop

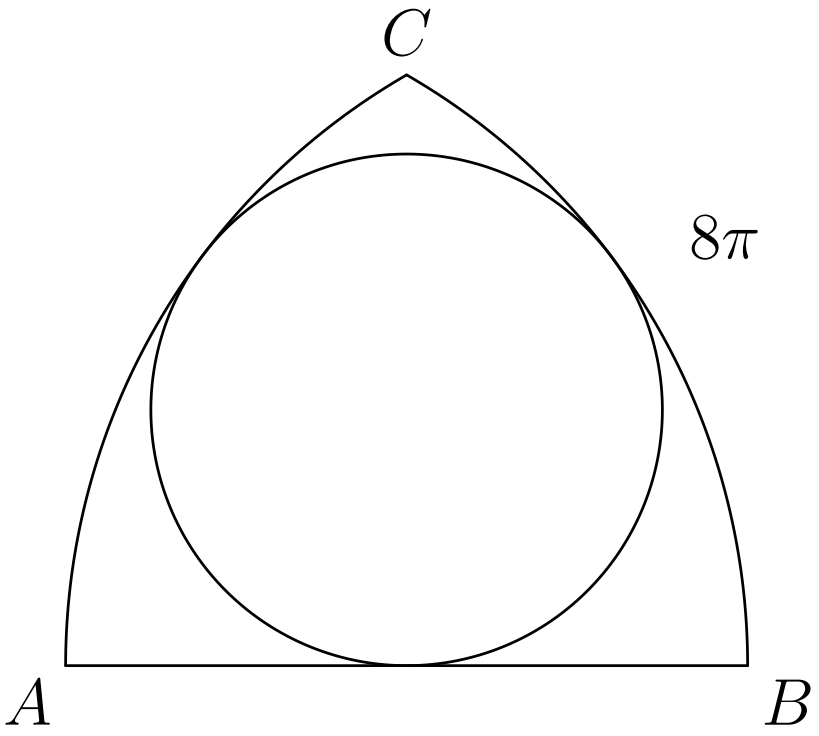
Answer

7. Let s denotes the side length of the largest equilateral triangle that is inscribed inside a square of side length 3. Find s^2.

Start Pause Stop

Answer

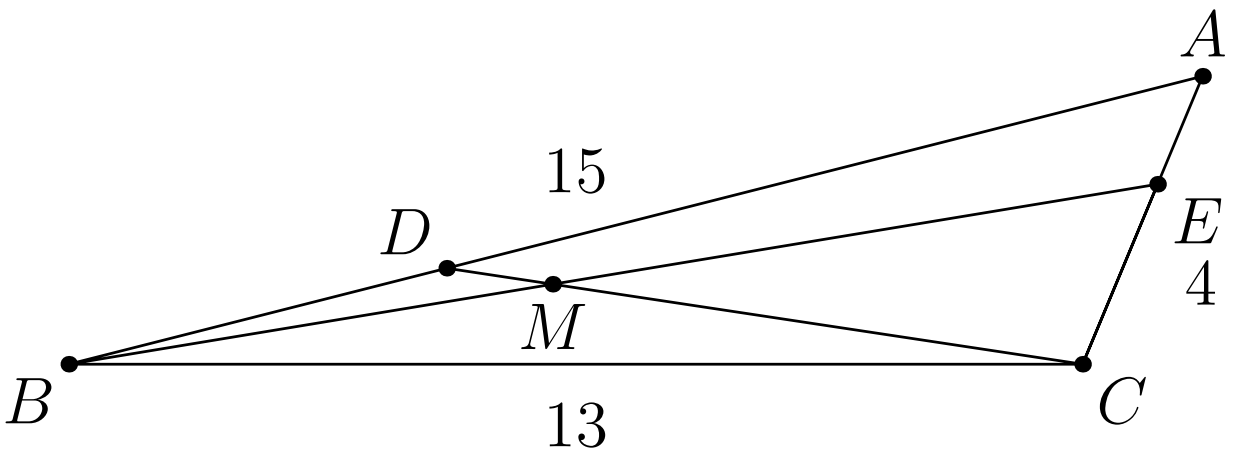
8. Circular arcs AC and BC have centers at B and A, respectively, and the circle in the figure is tangent to arcs AC and BC and to line AB. The length of arc BC is 8π. What is the area of the circle?



Start Pause Stop

Answer

9. Triangle ABC has $AB = 15$, $BC = 13$, and $AC = 4$. D and E are on AB and AC, respectively, such that $[DBC] = 8$ and $[EBC] = 15$. DC and EB intersect at M. Find $[ADME]$. Note that $[XYZ]$ denotes the area of polygon XYZ.

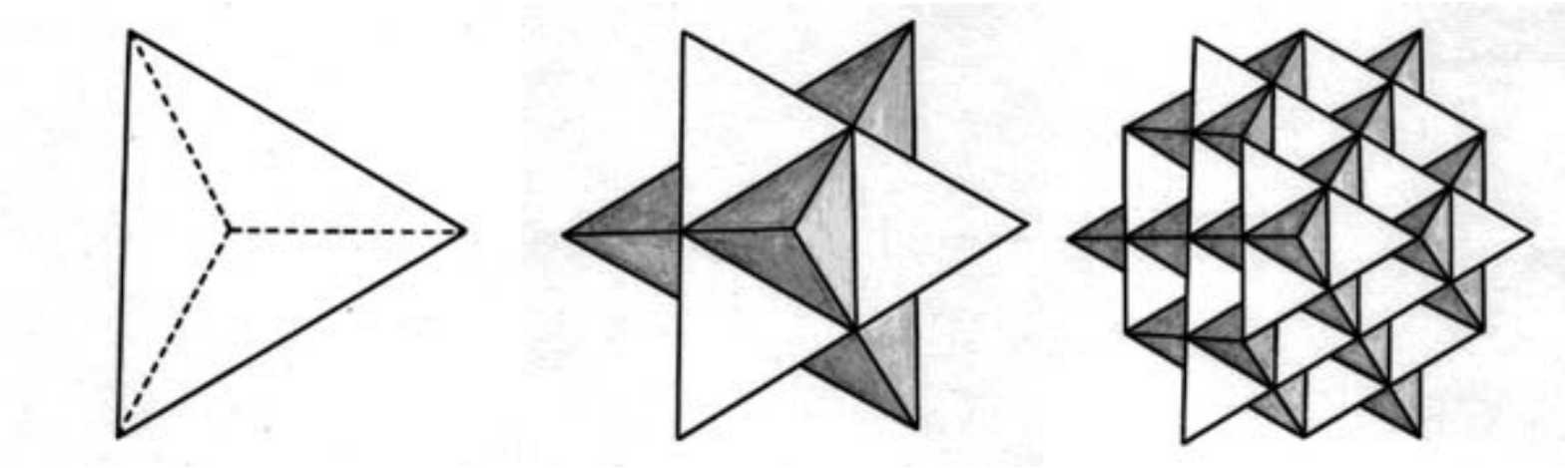


Start Pause Stop

Answer

10. Suppose I start with a regular tetrahedron with side length 2. In each step, I add a smaller regular tetrahedron onto every face such that the midpoints of each face are three of the vertices of the smaller tetrahedron. If I continue forever, what is the volume of the polyhedron?

The volume of a regular tetrahedron with side length s is $\frac{s^3\sqrt{2}}{12}$.



Bonus: What shape does this become?

Start Pause Stop

Answer

Round 3 Tiebreakers

You will have 3 minutes to solve one problem. When you solve a problem, raise your hand, and your proctor will grade it as well as assign you a time if you are correct. The participant with the lowest time moves on to the next round. In the case of both participants getting the problem wrong, we'll move on to a second problem, and so forth.

Go to End

1. A and B are points on a circle with center O and radius 8. Let P be the center of a circle internally tangent to circle O and tangent to OA and OB. Given that $AB = 3$, find the radius of P.

Start Pause Stop

Answer

Go to End

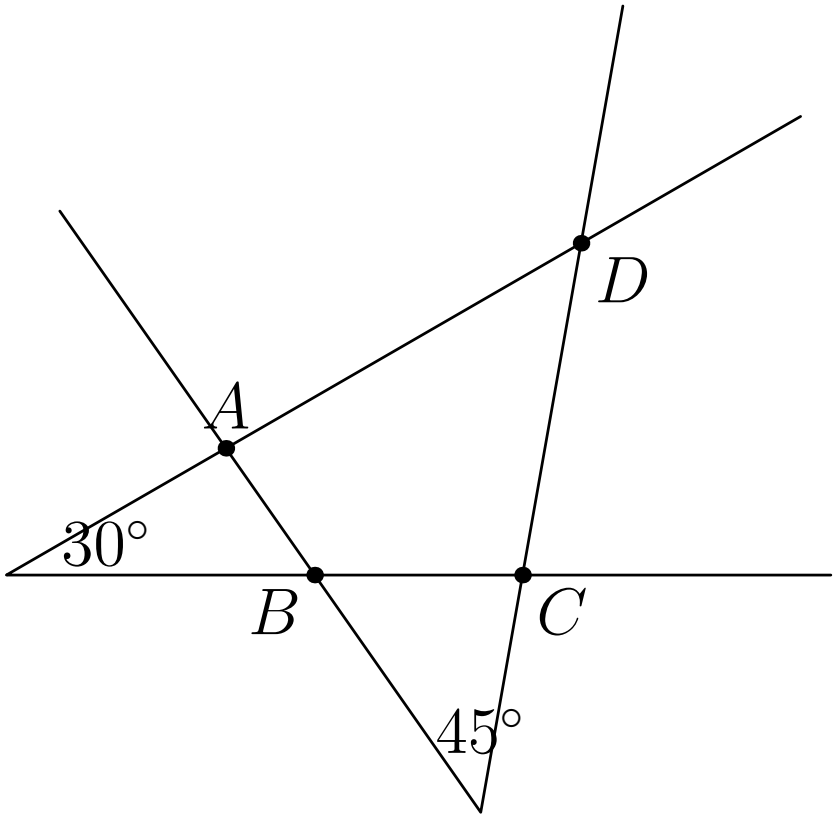
2. A circle is divided into two sectors, one with a central angle of $120°$, the other $240°$. Each is wrapped into a cone. What is the ratio of the volume of the larger cone to the smaller cone?

Start Pause Stop

Answer

Go to End

3. A 45° angle and a 30° angle overlap, as shown, creating quadrilateral $ABCD$. Find the difference between $\angle ABC$ and $\angle ADC$ in degrees.

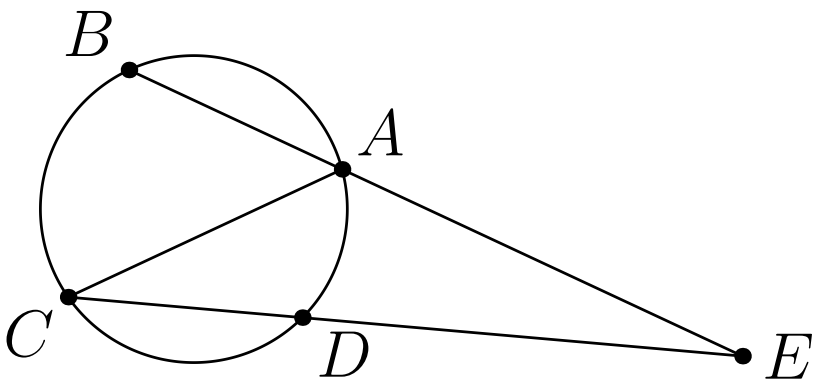


Start Pause Stop

Answer

Go to End

4. In the figure, angle E equals $20°$, and arc $AB = BC = CD$, what is $\angle ACD$?



Start Pause Stop

Answer

Go to End

5. Find the area of a right triangle with hypotenuse 25 and base 7.

Start Pause Stop

Answer

Go to End

Thank You for Coming!

Leftovers

Extra

1. A standard track is a total of 400 m long with 2 100 m straightaways and 2 semicircle turns. What is the area, in m^2, of the region contained by a track?

Start Pause Stop

Answer

2. Two identical spinning squares with side length 4 cm are pegged to a board such that their centers are 14 cm apart. Both squares begin aligned with a diagonal along the line running through their centers. A loop of string is tightly wrapped around both squares and turns along with them without slipping, causing the squares to spin in unison. A red dot is marked on the string at one of the corners on the axis connecting the squares. What distance does the dot travel before returning to its starting point?

NEEDS DIAGRAM

Start Pause Stop

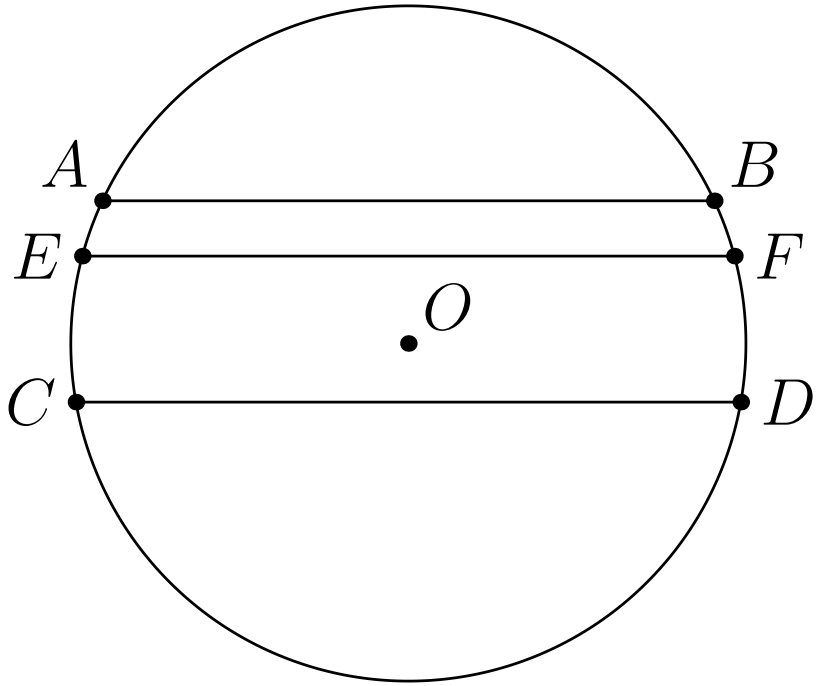
Answer

3. Lines l and m are walls lying parallel to each other. Lux stands on a point on line m, looking at line l making a 15° angle with line l. Katarina stands on line l to the right of Lux's vision, and she looks at line m making a 30° angle with line m, but she sees Lux. The distance is Katarina's vision, given that line m is a solid wall and Katarina cannot see past it, is $a \sin b$, where b is in degrees. Compute $a + b$.

Start Pause Stop

Answer

4. Two parallel chords AB and CD in a circle have lengths 12 and 16, and the distance between them is 6. A third parallel chord EF is between AB and CD. The perpendicular distance between AB and EF is 2. What is the length of EF?



Start Pause Stop

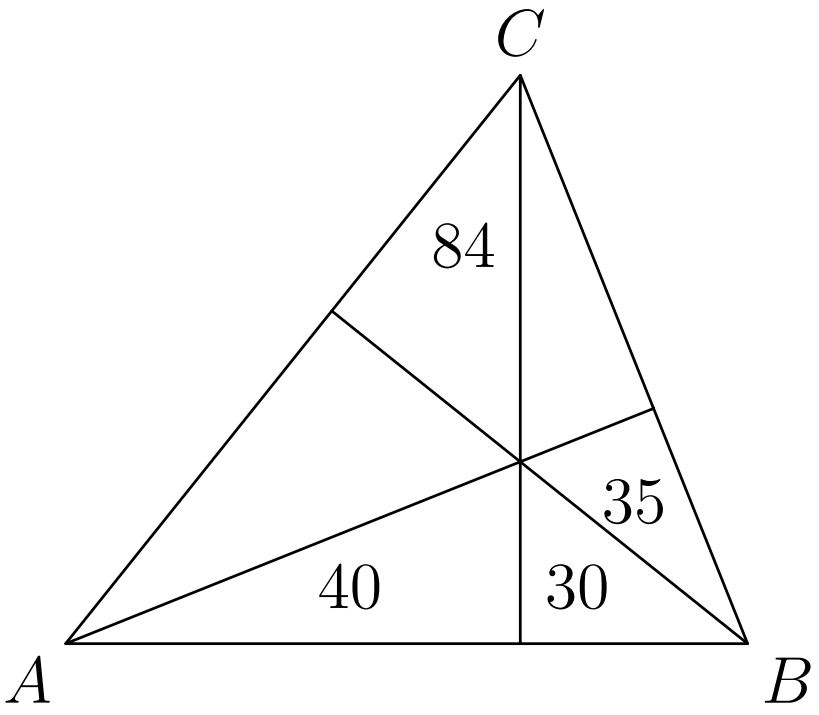
Answer

5. The side lengths of triangle ABC are $BC = 9$, $AC = 12$, and $AB = 15$. Lines 2 units away from the sides of triangle ABC are drawn, outside triangle ABC. Find the area of the triangle determined by these lines.

Start Pause Stop

Answer

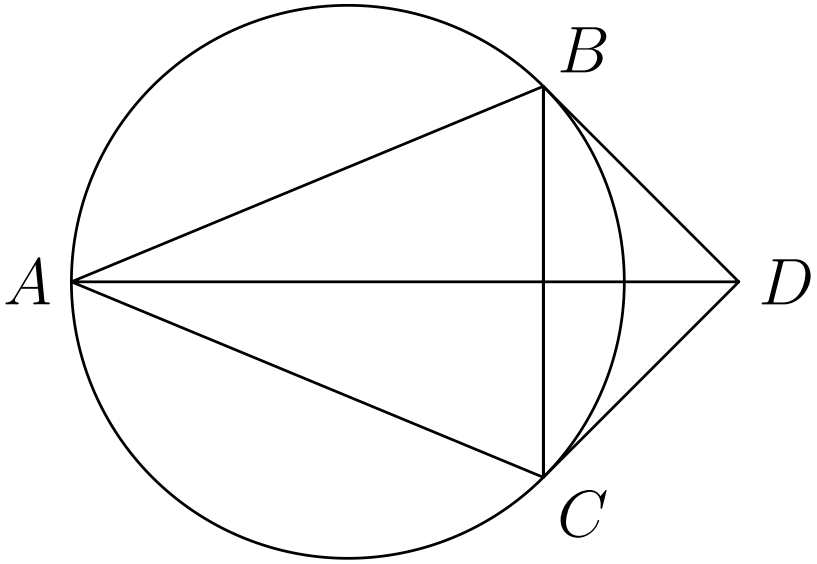
6. As shown in the figure, triangle ABC is divided into six triangles by lines drawn from the vertices through a common interior point. The areas of these triangles are as indicated. Find the area of triangle ABC.



Start Pause Stop

Answer

7. An acute isosceles triangle ABC is inscribed in a circle. Tangents to the circle are drawn at B and C, meeting at point D with $\angle ABC = \frac{2}{3}\angle BDC$. What is the radian measure of $\angle BAC$?



Start Pause Stop

Answer

8. Find the length of the inradius of a triangle with side lengths 7, 7, and 10.

Start Pause Stop

Answer

9. A triangle has vertices $P = (-8, 5)$, $Q = (-15, -19)$, and $R = (1, -7)$. The equation of the bisector of $\angle P$ can be written in the form $ax + 2y + c = 0$. Find $a + c$.

Start Pause Stop

Answer

10. In rectangle $ABCD$, points F and G lie on AB such that $AF = FG = GB$ and E is the midpoint of DC. Also, AC intersects EF at H and EG at J. The area of rectangle $ABCD$ is 140. Find the area of triangle AHF.

Start Pause Stop

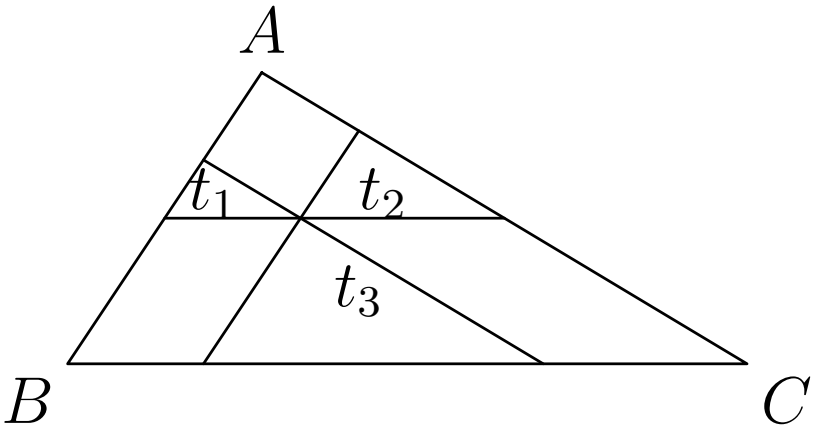
Answer

11. In triangle ABC, $AC = 12$, $BC = 5$, and $AB = 13$. The infinite sequence of points $C_1, C_2, C_3, C_4, \ldots$ is generated as follows: C_1 is the foot of the altitude from C to side AB, C_2 is the foot of the altitude from C_1 to side AC, C_3 is the foot of the altitude from C_2 to side AB, and so on. Calculate the sum $CC_1 + C_1C_2 + C_2C_3 + C_3C_4 + \cdots$.

Start Pause Stop

Answer

12. A point P that is chosen in the interior of triangle ABC such that when the lines are drawn through P parallel to the sides of triangle ABC, the resulting triangles t_1, t_2, and t_3 in the figure have areas 4, 9, and 49, respectively. Find the area of triangle ABC.



Start Pause Stop

Answer

13. Consider a cube $ABCDEFGH$ with side length 16 and AE as a space diagonal. At the center of each face on this cube, a square hole of side length 8 is drilled all the way to the opposite face. Brian the ant is located on vertex A, and can only travel along the surface of the cube. Compute the least distance Brian can travel to reach vertex E.

Start Pause Stop

Answer

14. A triangle is strong if all side lengths are integer length and it has two side lengths of 2013. How many strong triangles exist?

Start Pause Stop

Answer

15. Given that a regular polygon has at most 15 sides and the number of sides is randomly chose, what is the probability that this polygon will have integer values for angles, in degrees?

Start Pause Stop

Answer

16. Let ABC be a triangle such that $AB = 5$, $BC = 12$, and $CA = 13$. A laser starts at point B and shoots toward CA, bounces off and hits BC, and finally bounces off and ends in corner A. Compute the length of the distance this laser traveled.

Start Pause Stop

Answer